Solution exercise 3.4: Square Box

The energy levels are:

$$E_{n_1,n_2} = \frac{h^2}{8mL^2}(n_1^2 + n_2^2)$$

Where $n_1, n_2 = 1, 2, 3, \ldots$

Two different pairs (n_1, n_2) and (n_1', n_2') give the same energy if:

$$n_1^2 + n_2^2 = n_1'^2 + n_2'^2$$

The most obvious way this can happen is if you **swap** the two quantum numbers, since addition is symmetric.

For example:

$(n_1, n_2) = (1, 2)$ and $(2, 1)$ → both give $1^2 + 2^2 = 5$

First three degenerate cases:

$$E_{1,2} = E_{2,1}$$
$$E_{1,3} = E_{3,1}$$
$$E_{2,3} = E_{3,2}$$

So the degeneracy in the square box comes from the fact that the box is symmetric in both directions.

Solution exercise 3.5: Rectangular Box

Now the sides are not equal. Let's call them $L_1$and $L_2 = 2L_1$.

The energy levels become:

$$E_{n_1,n_2} = \frac{h^2}{8m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2}\right)$$

Substitute $L_2 = 2L_1$:

$$E_{n_1,n_2} = \frac{h^2}{8mL_1^2}\left(n_1^2 + \frac{n_2^2}{4}\right)$$

Multiply through by 4 to clear the fraction:

$$E_{n_1,n_2} = \frac{h^2}{32mL_1^2}(4n_1^2 + n_2^2)$$

So the energy depends on the combination $4n_1^2 + n_2^2$.

Condition for degeneracy:

Two different pairs give the same energy if:

$$4n_1^2 + n_2^2 = 4n_1'^2 + n_2'^2$$

One way this happens is if you "swap" values in a way that keeps this sum the same.

By trial and error, you can find:

- $(n_1, n_2) = (1,4)$ and$(n_1'^2, n_2'^2) = (2,2)$

$$For\ (1,4): 4(1)^2 + (4)^2 = 4 + 16 = 20$$

$$For\ (2,2): 4(2)^2 + (2)^2 = 16 + 4 = 20$$

Another valid solution is:

- $(n_1, n_2) = (1,6)$ and $(n_1'^2, n_2'^2) = (3,2)$

$$For\ (1,6): 4(1)^2 + 6^2 = 4 + 36 = 40$$

$$For\ (3,2): 4(3)^2 + (2)^2 = 36 + 4 = 40$$